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Constraint Supersymmetry Breaking and Non-Perturbative Effects in String Theory

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Abstract

We discuss supersymmetry breaking mechanisms at the level of low energy $\mathcal{N}=1$ effective heterotic superstring actions that exhibit $SL(2,Z)_T$ target space modular duality or $SL(2,Z)_S$ strong-weak coupling duality. The allowed superpotential forms use the assumption that the sourse of non-perturbative effects is not specified and as a result represent the most general parametrization of non-perturbative effects. We found that the allowed non-perturbative superpotential is severely constrained when we use the cusp forms of the modular group for its construction. By construction the poles of the superpotential are either inside the fundamental domain or beyond. We also found limits on the parameters of the superpotential by demanding that the truncated potential for the gaugino condensate never breaks down at finite values in the moduli space. The latter constitutes a criterion for avoiding poles in the fundamental domain. However, the potential in most of the cases avoids naturally singularities inside the fundamental domain, rendering the potential finite. The minimum values of the limits on the parameters in the superpotential may correspond to vacua with vanishing cosmological constant.

1 Introduction

One of the biggest problems that heterotic string theory, and its "equivalents", e.g type II, I, have to face today is the question of $\mathcal{N}=1$ space-time supersymmetry breaking. The breaking, due to the presence of the gravitino, that determines the scale of supersymmetry breaking, in the effective action, must be spontaneous and not explicit. Several mechanisms have been used in recent years to break consistently supersymmetry. They can distinguished as to when they are at work at the string theory level or at the effective superstring action level. The first category of mechanisms includes the tree level coordinate dependent compactification mechanism [1], the magnetized tori approach [3], the type I brane breaking [4, 5, 6], the partial breaking [7] while the latter category includes approaches that use target space duality e.g [9, 15, 10, 16] or S-duality [21, 28] at the level of effective superstring action, related to gaugino condensation [8], to constrain the allowed superpotential forms. The main problem in all approaches is the creation of an appropriate potential for the moduli and the dilaton that can fix their vacuum expectation values. The first category was made popular quite recently because of our understanding of non-perturbative effects in string theory via the discovery of D-branes, oblects where open strings can end. However, we might not forget that gaugino condensation, a non-perturbative field theoretical effect can break supersymmetry, satisfactory espacially when two condensates are used [41]. In fact breaking supersymmetry by gaugino condensation (GC) has its advantages. In fact if we knew the non-perturbative contribution to the gauge kinetic function, GC would have been an excellent mechanism and we wouldn't have to look on string mechanisms to break supersymmetry. Here, we would do exactly that. Because heterotic string theory has target space duality as one of its properties we can use modular forms to parametrize the unknown non-perturbative dynamics [16], practically to parametrize the unknown non-perturbative contributions to the gauge kinetic function f. Something similar could not be done e.g for type I strings as they don't possess target space duality so a string breaking of supersymmetry for the latter may be the most appropriate.

The purpose of this paper is to reexamine the issue of constructing superpotentials W^{np} that affect supersymmetry breaking at the level of $\mathcal{N}=1$ effective heterotic superstring actions when the sourse of non-perturbative effects, is not specified. We will not perform a full a numerical study of the scenarios proposed as we leave it for future work. In particular in this work, we examine and improve in earlier scenaria [21, 16, 28] the assumptions used in the construction of W^{np}.

The modification of the $\mathcal{N}=1$ heterotic effective action that we examine in

this work amounts to modifying the superpotential when T-duality or S-duality non-perturbative effects are included. In this work we want to break supersymmetry dynamically, rather than geometrically, thus we make use of gaugino condensation in supergravity [36]. In general there are two different approaches in describing gaugino condensation. These are the effective lagrangian approach [9, 10] where we can use a gauge singlet bilinear superfield U as a dynamical degree of freedom and the effective superpotential approach [15, 16]. In the latter formalism the gauge singlet bilinear superfield is integrated out through its equation of motion.

The paper is organized as follows. In section 2 we review the current status of the most general parametrization of non-perturbative effects into the vacuum structure of $\mathcal{N} = 1$ heterotic superstrings through modifications of the superpotential in the effective superpotential approach. In section 3 we describe new parametrizations of non-perturbative effects by constructing the most general weight zero superpotential factor invariant under $SL(2, Z)$ modular transformations. It is found, by using cusp modular forms that the resulting superpotential automatically includes weak coupling limit constraints in its form, when it is used to describe S-dual superpotentials.

Moreover we connect previously unrelated constructions with vanishing cosmological constant and broken supersymmetry in the dilaton sector, auxiliary dilaton field h_s non-zero [16], to our non-perturbative superpotential constructions through basis for modular forms.

In section 4 we decsribe the most general modifications to the superpotential in the effective lagrangian approach of [35] when T-duality non-perturbative effects are included with [39, 40] and without the formation of matter field condensates [35, 36]. In section 4.3 we test the stability of the condensate dynamics for the new superpotential constructions. We found strong constraints on the parameters of the superpotential such that the truncated approximation never breaks down as the potential approaches its self-dual points. We found that the lower classes of superpotentials with finite potential at finite points in the upper half plane may coincide with the vacua with vanishing cosmological constant mentioned in section 3. Finally, in section 5 we present our conclusions and some future directions of this work.

2 Allowed forms of non-perturbative superpotentials

The effective supergravity theory coming from superstrings is described by the knowledge of three functions, the Kähler potential, the superpotential W and the gauge kinetic function f, that all depend on the moduli fields. For $(2,2)$ heterotic string compactifications with $\mathcal{N}=1$ supersymmetry there is at least one complex modulus which we denote by $T=R^2+ib$, where R is the breathing mode of the six dimensional internal space and b is the internal axion θ. The T-field corresponds, when T large, to the globally defined $(1,1)$ Kähler form. Here we will restrict our study to the simplest (2, 2) models where there is a single overall modulus, by freezing all other T-moduli. In the gravitational sector the lowest component of the chiral dilaton superfield S forms a complex scalar modulus, combining the gauge coupling constant as its real part with the pseudoscalar axion field,

$$S=\frac{1}{g^2}+i\theta. \tag{2.1}$$

The part of the Kähler potential the includes the tree level contribution[1] to the dilaton is

$$K(S,\bar{S})=-\log(S+\bar{S}). \tag{2.2}$$

The Kähler potential, for the T-field, is defined as

$$K(T,\bar{T})=-\log(T+\bar{T})^3 \tag{2.3}$$

in its tree level form and

$$K(T,\bar{T})=-\log\{(T+\bar{T})^3+\mathcal{I}_{instanton}\} \tag{2.4}$$

in the presence of instantons.

The first term in (2.4) dominates in the large radius limit, in the σ-model sense, and can be derived from the field theoretical truncation limit of 10D $\mathcal{N}=1$ heterotic string. The second term represents the contribution of the non-perturbative effects and is associated with instantons.

Because moduli fields have flat potential to all orders of perturbation theory [11] their vacuum expectation value's (vev's) remain undetermined. As a result the task of

[1]For simplicity we neglect the Green-Schwarz term contribution as it does not introduce any additional dilaton dependence on the Kähler potential [25]. Since there is no known method to calculate the non-perturbative corrections to the Kähler potential, we consider the Kahler potential as receiving its tree level value.

lifting their degenerate vev's is attributed to non-perturbative effects. In the absence of a mechanism of calculating non-perturbative corrections to the Kähler potential we can choose to include a general parametrization of T-duality effects in the non-perturbative superpotential that includes the dilaton. Furthermore we assume that the T-modulus and the dilaton dependence in the superpotential factorize. One further constraint in the form of allowed superpotentials comes from its modular weight and the presence of physical singularities in the upper half plane. All the previous information can be used to construct non-trivial modular superpotentials with T or S moduli, that can give information about the general dynamics of superstring vacua. The origin of the perturbative terms in the superpotential may be understood in the context of the orbifold limit of $\mathcal{N} = 1$ four dimensional F-theory compactifications on a four-fold $CY_4(CY_4 = (CY_3 \times T^2)/Z_2)$ dual to $(0,2)$ heterotic string compactifications on a three-fold Z over a base $F_k(Z = (K_3 \times T^2)/Z_2$, with a choise of gauge bundle on $E_8 \times E_8$. In this case, assuming that no heterotic 5-branes, the F-theory superpotential [12], with no 3-branes present, can match the perturbative heterotic superpotential [13]. The non-perturbative contributions originate from the introduction of type IIB three branes in F-theory necessasy for cancelling tadpole matching of heterotic five-branes(equivalently we may consider four dimensional M-theory compactifications [14] on Z $\times S^1/Z_2$, namely compactifications on a Calabi-Yau threefold Z with vector bundle Z embedded in $E_8 \times E_8$.). The latter objects are such that their four dimensional part spans the four dimensional Minkowski space, while the two remaining wrap around a holomorphic curve in Z. However, in the non-perturbative case the question still remains how we can calculate explicit non-perturbative superpotentials that translating them in heterotic language we can derive general conclusions about the potential vacuum structure of heterotic strings.

Because the effective supergravity action of the heterotic string is invariant under the target space duality transformations, that holds in all orders of perturbation theory [17]

$$T \to \frac{AT - iB}{iCT + D},\ AD - BC = 1, \tag{2.5}$$

since the $G = K + \log|W|^2$ function has to remain invariant, we obtain that the superpotential has to transform with modular weight -3 [18]. Note that if the S-duality principle [21, 22, 23, 24] is proved to be valid principle at the level of $\mathcal{N} = 1$ heterotic effective actions the effective superstring action may be invariant under the $SL(2,S)_S$ transformations

$$S \to \frac{A'S - iB'}{iC'S + D'},\ A'D' - B'C' = 1, \tag{2.6}$$

and the superpotential has to transform with modular weight -1 under (2.6).

In [16], based on a mathematical theorem of modular forms [19], the most general holomorphic modular function[2] of weight r, for a moduli Φ, was written in the form

$$[G_6(\Phi)]^m [G_4(\Phi)]^n [\eta(\Phi)]^{2r-12m-8n} \mathcal{P}(j(\Phi)), \tag{2.7}$$

or equivalently, the superpotential W,

$$W(\Phi) = (j - 1728)^{m/2} j^{n/3} [\eta(\Phi)]^{2r} \mathcal{P}(j(\Phi)), \tag{2.8}$$

$$W(\Phi) = \Omega(\Phi) [\eta(\Phi)]^{2r} \mathcal{P}(j(\Phi)), \tag{2.9}$$

$$\Omega(\Phi) = (j - 1728)^{m/2} j^{n/3}, \tag{2.10}$$

where m, n positive integers and G_6 , G_4 are the Eisenstein functions of modular weight six and four and $\mathcal{P}(j(\Phi))$ an arbitrary polynomial of the absolutely modular invariant $j(\Phi)$. Depending on whether the superpotential has modular weight -3 as it is the case of a T-duality invariant superpotential or modular weight -1 as it is the case of an S-duality invariant superpotential the classes of superpotentials in (2.7-2.10) were written in the following forms when r equals -3 [16]

$$W(T, S) = \frac{(j(T) - 1728)^{m/2} j^{n/3}(T)}{\eta^6(T)} \mathcal{P}(j(T)) \mathcal{K}(S), \tag{2.11}$$

where $\mathcal{K}(S)$ parametrizes the dilaton dynamics, or -1 [21] respectively,

$$W(S) = \frac{(j(S) - 1728)^{m/2} j^{n/3}(S)}{\eta^2(S)} \mathcal{P}(j(S)). \tag{2.12}$$

In the last equation we have chosen not to exhibit its T-dependence. The behaviour of the superpotentials in (2.11), (2.12), is such that the potential diverges when $T, S \to \infty$, respectively. However, this runaway behaviour is avoided as duality stabilizes the potential at finite points.

In general duality stabilizes the potentials with local minima at the points $T, S = 1, \rho$. The minima on the cases considered in [21, 16] are either at the self-dual points giving unbroken space-time supersymmetry in the T, S field sector respectively or in the general case supersymmetry breaking minima with negative cosmological constant. In the latter case the minima occur at the boundary of the moduli space.

[2]which has no singularities in the fundamental domain

It is worth mentioning at this point that (2.11) is equivalent [16] to defining[3]the gauge kinetic function in the form

$$f = S - \frac{|G_i|}{|G|}(b_a^{N=2} \log[\eta^4(T)(T+\bar{T})] - \frac{1}{16\pi^2} Re\{\partial_T \partial_U h^{(1)}(T,U) - 2\log((j(T)-j(U))\}) + (b_a/3)\log|\Omega(T)|^2 + \mathcal{O}(e^{-S}), \tag{2.13}$$

where $h^{(1)}$ the one-loop prepotential[4], G_i the orders of the subgroup G which leaves the i-complex plane unrotated and we have included the one-loop Green-Schwarz term [31]. Note however, that in (2.11) we have neglected the contribution from the second term of (2.13) as it is not needed in our present study. Its contribution will be examined elsewhere.

The unknown dilaton dynamics, parametrized by $\mathcal{K}(S)$ in (2.11), involves the contribution from the tree level dilaton term and the non-perturbative dilatonic contributions of (2.13).

Another attempt to study the classes of superpotentials of eqn.'s (2.7-2.10) was made in [28]. In [28] the behaviour of this class of superpotentials was examined in the context of S-duality of $\mathcal{N}=1$ heterotic string effective action. The authors used (2.10), when $r=-1$, by imposing in addition, what they called, "validity" of weak coupling perturbation theory. The latter condition means that the superpotentials are regular anywhere except the $S \to \infty$ limit where they can have singularities. That enforces the condition that the allowed forms of $W(S)$ may be in the form[5]

$$W(S) = \left(\frac{1}{\eta(S)^2}\right)\left(j^{n/3}(z)(j-1728)^{m/2}(S)\right)\left(\frac{P_1(j)}{P_2(j)}\right), \tag{2.14}$$

or equivalently

$$W(S) = \left(\frac{1}{\eta(S)^2}\right)(\Omega'(S))\left(\frac{P_1(j)}{P_2(j)}\right), \tag{2.15}$$

$$\Omega'(S) = \left(j^{n/3}(S)(z)(j(S)-1728)^{m/2}\right), \tag{2.16}$$

where m, n positive integers and $degP_2 > degP_1 + \frac{1}{3}n + \frac{1}{2}m + \frac{1}{12}$, and P_1, P_2 are polynomials in j.

[3] in the following section $\Omega(T)$ will be replaced by the more general form $\Sigma(T)$

[4] The one loop $\mathcal{N}=2$ four dimensional vector multiplet prepotential $h^{(1)}$ was calculated as an ansatz solution to a differential equation involving one loop corrections to gauge coupling constant in [26]. However, its exact general form for any four dimensional compactification of the heterotic string was calculated in [32]. Higher derivatives of $h^{(1)}$ were also calculated in [27].

[5] we express only the S-part of the superpotential

The class of superpotentials (2.16) have negative cosmological constant at its minimum $S = 1$. Moreover, the behaviour of (2.14), since W is a section on a flat holomorphic line bundle over the moduli space of $SO(2) \setminus SL(2,R)/SL(2,Z)$, fixes from the behaviour of $\Omega(S)$ under the dilatational transformation $S \rightarrow S+1$ near $S \sim i\infty$,

$$n \; mod \; 3, \quad m \; mod \; 2. \tag{2.17}$$

That means that Ω' is constrained to be

$$\Omega'_{cons}(S) = [\Big(j^{n/3}(S)(j(S)-1728)^{m/2}\Big)]|_{n \; mod \; 3, \; m \; mod \; 2} \tag{2.18}$$

Our first reaction by looking at the factor Ω', of (2.16) which is a part of S-duality invariant superpotentials and Ω of (2.10), that describes part of a T-duality invariant superpotential, may be that they are quite different as the range of parameters in the exponentials are different. Ideally, we would expect that given the Ω', Ω to be quite the same since they have both the same modular weight zero. In addition, because (2.11) should transform as a line bundle under target space duality translational transformations at infinity, we should expect the constraints (2.17) to be apriori incorporated even on the T-duality superpotentials (2.11).

In the next section, we will suggest that this is the case, $\Omega \rightarrow \Omega'_{cons}$, and the constaints (2.17) may be incorporated apriori in the superpotentials of (2.11) or (2.12).

We recall [16] one more result, in the context of T-duality transforming superpotentials namely that it is possible to construct superpotentials $W^j(T,S)$ that break supersymmetry and have vanishing cosmological constant at generic points in the moduli space, when the auxiliary field h_s is not zero, in the form

$$W^j(T,S) = \frac{\mathcal{K}(S)}{\eta^6(T)} \Big(j(T)^3 + l\, j(T)^2 + m\, j(T) + n\Big), \tag{2.19}$$

where $\mathcal{K}(S)$ decsribes the unknown dilaton dynamics and l, m, n constants determined by the vacuum dynamics. The superpotentials (2.19) have in addition minima with unbroken supersymmetry and negative cosmological constant at the self-dual points $T = 1$, $h^T = 0$. In the following section we write down a more general class of superpotentials that incorporates parts of approaches of [16, 21, 28] as well connecting the factor Ω'_{cons} to the class of superpotentials (2.19).

3 New constructions of non-perturbative superpotentials

3.1 *SL(2,Z)'s modular group transforming W's*

The method of using (2.7) as a starting point to construct superpotential W's that parametrize the unknown dynamics works only if the modular weight r is not zero. In addition the superpotential (2.11) misses the constraint (2.17). Here, we follow a different approach such that (2.17) is included by construction in the superpotential. First we demand that the behaviour of the modular weight factor[6] Ω factorizes from the overall[7] factor η^{2r}. That means that it is impossible to describe the superpotential in terms of its form (2.10). That happens because the construction of (2.10) was based on a mathematical theorem of modular forms that interconnects the factor η^{2r} to the other factors in front involving G_4, G_6. Our objective in this paper is to find a way of producing the class of superpotentials of (2.18) in such a way such that the constraints of the monodromy behaviour of W at infinity are included apriori.

Our construction proceeds by writing an analogous expression to (2.7). What we want to construct is the most general modular factor which transforms with modular weight zero under SL(2,Z) modular transformations. After we have done that we can use this factor to discuss how we can construct superpotentials with or without singularities in the upper half complex plane.

The clear difference in our constructions, against the approaches of [16, 21, 28], is that do not treat the η-invariant as a fundamental quantity but rather the cusp forms $\triangle(z)$ of the modular group $SL(2,Z)$ instead. For this reason we write down the most general weight zero factor as

$$\tilde{\Sigma}(\Phi) = \frac{E_4^i(\Phi) E_6^j(\Phi)}{\triangle(\Phi)^k}, \tag{3.1}$$

where i, j, k arbitrary integers. In order that the modular factor $\tilde{\Sigma}(\Phi)$ to transform with modular weight zero the following condition must hold,

$$4i + 6j = 12k, \tag{3.2}$$

with solution

$$i = 3a, \quad j = 2b, \quad a, b \in Z \tag{3.3}$$

Applying the constraint (3.3) to eqn. (3.1) we get

$$\tilde{\Sigma}(\Phi) = \frac{E_4^{3a}(\Phi)}{\triangle(\Phi)^a} \frac{E_6^{2b}(\Phi)}{\triangle(\Phi)^b} \tag{3.4}$$

[6]in real terms a modified version of it

[7]Note that the overall factor η^{2r} is associated with the one loop string threshold corrections to the gauge coupling constants [20].

or equivalently

$$\tilde{\Sigma}(\Phi) = \left(\frac{E_4^3(\Phi)}{\triangle(\Phi)}\right)^a \left(\frac{E_6^2(\Phi)}{\triangle(\Phi)}\right)^b = j^a \cdot (j - 1728)^b, \tag{3.5}$$

where a, b integers and j the SL(2,Z) modular function.

The non-perturbative superpotential, that includes the dilaton can now be written into the form

$$W^{new}(T,S) = \frac{\mathcal{K}(S)\,\tilde{\Sigma}(T)}{\eta^6(T)}\mathcal{P}(j(T)), \quad a,b \in Z, \tag{3.6}$$

where $\mathcal{P}(j(T))$ an arbitrary polynomial of the absolute modular invariant j.

Note that the factor $\tilde{\Sigma}(T)$ that has modular weight zero parametrizes the unknown T-modulus dynamics and incorporates apriori by construction the constraints (2.17). However, there is an extra degree of freedom in the way that we could define a candidate T-dual superpotential since the following superpotentials have the same modular weight, and are both allowed

$$W(T) = \frac{\tilde{\Sigma}(T)}{\eta^6(T)}\mathcal{P}(j), \tag{3.7}$$

$$W(T) = \frac{1}{\eta^6(T)}\frac{1}{\tilde{\Sigma}(T)}\mathcal{P}(j). \tag{3.8}$$

However, this is a novel feature of our construction since the candidate superpotentials (3.8) solve the decompactification problem of the potential (3.7)(which tends to ∞) since they are finite at this limit and go to zero.

One more observation should be added here. Defining the candidate T-dual superpotentials into the forms (3.7), (3.8) is equivalent to demanding that do not allow or do allow poles in the upper half plane respectively. The associated scalar potentials are such when $T \to \infty$, $V \to \infty, 0$ respectively. That happens because modular functions which are allowed to have poles in upper half plane are exactly rational functions in j (quotients of polynomials in j) whereas modular functions which are not allowed to have such poles are polynomials in j.

Let us now discuss if there is anyway that string vacua described by the superpotential (3.5) are connected in anyway to the class of solutions (2.19) of [16]. The answer ot this question comes from the theory of modular forms. The expression (2.19) is a special basis of expressing a modular zero form in terms of a polynomial in j(z) instead of using the more obvious basis $j(z)^n$, $n = 0, 1, 2, \cdots$ in the space of polynomials of $j(z)$. *That means that the expressions of eqn.'s (3.5) and (2.19) represent expressions for superpotentials expressed in different basis for modular forms.* The question as to

whether string theory can fix exactly[8] the parameters a, b in (3.5) has to be decided when a non-perturbative calculative framework in the heterotic string context is found. The vacuum structure of the superpotentials (3.7) may be examined by the study of the effective scalar potential V,

$$V = |h_s|^2 G^{-1}_{S\bar{S}} + |h_T|^2 G^{-1}_{T\bar{T}} - 3exp(G) = \tag{3.9}$$

$$= \frac{1}{S_R T_R^3 |\eta(T)|^{12}} \left(|S_R \mathcal{K}_S - \mathcal{K}|^2 |\tilde{\Sigma}|^2 + \frac{T_R^2}{3} |\tilde{\Sigma}_T + \frac{3}{2\pi} \tilde{\Sigma} \tilde{G}_2|^2 |\mathcal{K}|^2 - 3|\mathcal{K}|^2 |\tilde{\Sigma}|^2 \right), \tag{3.10}$$

where $h_i = exp(G/2)$ the auxiliary field for the i-modulus. We will not attempt to perform an extensive numerical analysis of the above potential as this will be left for future work. We can however borrow some of the results of [16] which are inside our range of parameters and make some comments. In the case $\mathcal{P} = 1$, for $(a, b) = (0, 0)((m, n) = (0, 0))$, that is the case of [15], the minimum is at $T_{min} \approx 1.2$ while for $(a, b) = (0, 1)$ $((m, n) = (0, 3))$, the minimum appears is at the self-dual point $T_{min} = 1$ with the potential being negative and the other minimum is at the self-dual point $T = \rho = e^{i\pi/6}$ with zero potential.

Notice that the previously mentioned values have been calculated under the condition $S_R \mathcal{K}_S - \mathcal{K} = 0$, $S_R = (S + \bar{S})$, which makes the minima to occur for weak coupling, at large S_R.

Note the novel feature of (3.8) that when used to calculate the scalar potential makes it not to diverge and tend to zero when T goes to infinity.

3.2 *N = 1 Strong-weak coupling* $SL(2, Z)_S$ *superpotentials*

At this point we may make some comments related to how we could construct the most general $\mathcal{N} = 1$ four dimensional superpotentials that transform under the S-duality transformations (2.6). Our approach generalizes the constructions of classes of superpotentials in [21, 28]. Because of its construction the modular weight zero factor $\tilde{\Sigma}(S; a, b \in Z)$ has arbitrariness as to whether a, b are positive or negative integers or addressed in a different form as to whether our theory may have [21] or not singularities in the upper half-plane or at infinity [28]. We can choose for convenience the effective action of a possible $\mathcal{N} = 1$ S-dual heterotic string action.

In this case we might as well have as candidates dilaton dependent superpotentials,

[8] a lower limit on (a, b) may be provided in section 4.3

the following[9]

$$W(S)^{(1)}_{dual} = \frac{1}{\eta^2(S)}\tilde{\Sigma}(S)P(j(S)),\ a, b\ \in\ Z^{\dagger}, \tag{3.11}$$

or

$$W(S)^{(2)}_{dual} = \frac{1}{\eta^2(S)}\frac{1}{\tilde{\Sigma}(S)}P(j(S)),\ a, b\ \in\ Z^{\dagger}. \tag{3.12}$$

Both (3.11), (3.12) can be classified according as to whether we demand our superpotentials may have or not poles in the upper half plane, equivalently when the potential goes to zero or diverges when $S \to \infty$ respectively. In both cases the potential is finite for finite values of S. In the case of (3.11) there are no poles in the upper half-plane and the potential diverges when $S \to \infty$, while in (3.12) the potential goes to zero when $S \to \infty$. Thus the classes of superpotentials (3.12) resolve naturally, the problem of infinite potential of (3.11), when $S \to \infty$.

The superpotential (3.11) is different from the classes of superpotentials of [21], where the constraint (2.17) was not included, as the constraint (2.17) in our case is included by construction.

Let us make some comments at this point regarding the classes of superpotentials (3.12). At first look (3.12) appears to be equivalent to the classes of S-dual superpotentials (2.16) of [28]. However, (3.12) is more general. That happens partially because the constraints (2.17) are included by construction in its form and secondly because the existence of singularities in the upper half plane at the, large dilaton limit, weak coupling region, is enforced naturally by construction, e.g when $\mathcal{P}(j(S)) = 1$.

4 Gaugino condensation and supersymmetry breaking

The plan of this section is as follows. In subsect. 4.1 we reexamine the gaugino condensation approach without the use of matter fields in the effective lagrangian approach. In subsect. 4.2 we reexamine gaugino condensation in the presence of matter fields, finding all allowed forms of truncated superpotentials. In subsect. 4.3 we examine the stability of truncated superpotentials of subsect. 4.1, obtaining numerical constraints on the parameters a, b that are involved in the construction of the modular factor $\tilde{\Sigma}$ of (3.5).

[9] $Z^{\dagger}$ the space of positive integers

4.1 *Without matter field condensates*

We reexamine in this section earlier results [35, 36] on gaugino condensation using the effective lagrangian approach in effective supergravity theories from superstrings. In particular we will examine the way that the superpotential of the gaugino dynamics may be modified when the source of non-perturbative fields of the T-field is not specified, and defined via (3.5).

We suppose that the gauge group of the $\mathcal{N}=1$ supersymmetric superstring vacuum is a product of non-abelian gauge factors G_a, $a=1,\ldots,p$ and the gauge group is given by $G=\oplus\Pi_a G_a$. The effective local lagrangian that incorporates p-gaugino condensates is then given by

$$\mathcal{L}=-[\frac{1}{2}e^{-\frac{K}{3}}S_o\bar{S}_o]_D+[S_o^3 w]_F+(f_{ab}W^aW^b)_F, \tag{4.1}$$

where K is the Kähler potential, W is the superpotential and f_{ab} the gauge kinetic function. The effective action for the gaugino composite is described by defining chiral composite superfields U, Y such that

$$Y_n^3=\frac{(\delta_{ab}W_\alpha^a\epsilon^{\alpha\beta}W_\beta^b)_n}{S_o^3}=\frac{U}{S_o^3}, \tag{4.2}$$

where the scalar components of Y_n's are associated with the gaugino condensate $(\lambda\bar{\lambda})_n$ and the $SL(2,Z)_T$ modular weight of Y_n is -1. The choise of the chiral compensator S_o is such that it determines the normalization of the gravitational action $\mathcal{L}_{grav}=(16\pi^2)^{-1}M_p^2R$ with $[e^{-K/3}S_o\bar{S}_o]_{\theta=\bar{\theta}=0}=1$. The Kähler potential is given by

$$K=-3\log\left((T+\bar{T})(S+\bar{S})^{1/3}-\sum_{n=1}^{p}|Y_n|^2\right), \tag{4.3}$$

while the superpotential [36] is given by

$$w=\frac{1}{32\pi^2}\sum_{n=1}^{p}c_nY_n^3\log[Y_n^3\Psi_n(S)H_n^{(o)}(T)]. \tag{4.4}$$

In (4.4) we choose to fix the value of the dilaton by using more than one condensates That makes sure that a minimum at weak coupling could be found.

Because of anomalous Ward identities, under the $SL(2,Z)_T$ modular transformations (2.5),

$$Y_n\rightarrow\frac{1}{iCT+D}Y_n,\quad H_n^{(o)}\rightarrow(iCT+D)^3H_n^{(o)}, \tag{4.5}$$

while

$$\Psi_n(S)=e^{\frac{32\pi^2k_nS}{c_n}}. \tag{4.6}$$

The use of $H_n^{(o)}(T)$ is such that it compensates for the lack of modular invariance in the logarithmic term of (4.4). The function $H_n^{(o)}(T)$ were given a first estimate in [35] as

$$H_n^{(o)}(T) \propto \eta^6(T) \ , \tag{4.7}$$

by demanding absence of singularities in the effective action in the upper half-plane for complex T. We can now revise the estimate of $H_n^{(o)}$ in [35] by listing all possibilities allowed depending on what kind of singularities we admit to appear in the upper half plane in the truncated[10] superpotential W^{tr}.

We distinguish two cases:

$$H^{(o)} \rightarrow H_n^{(1)}(T) = \eta(T)^6 \frac{1}{j^a(T)(j(T)-1728)^b}; \ \ a, b \ \in \ Z^{\dagger}, \tag{4.8}$$

which causes W^{tr} to be regular in the upper half plane, and

$$H^{(o)} \rightarrow H_n^{(2)}(T) = \eta(T)^6 j^a(T)(j(T)-1728)^b; \ \ a, b \ \in \ Z^{\dagger}, \tag{4.9}$$

which allows W^{tr} to have poles in the upper half plane. The modular invariant scalar potential of the theory is given by

$$V(S,T,Z_n) = \{32\pi^2(1-\sum_{n=1}^{p}|Z_n|^2)^{-2}\} \cdot (V_Z + V_S + V_T - 3|\sum_{n=1}^{p} c_n Z_n^3|^2), \tag{4.10}$$

where

$$Z_n = (S+\bar{S})^{-1/6}(T+\bar{T})^{-1/2} Y_n \tag{4.11}$$

$$V_Z = 3\sum_{n=1}^{p} c^2 |Z_n|^4 \cdot |1 + \log\{(S+\bar{S})^{1/2}(T+\bar{T})^{3/2} f_n H_n Z_n^3\}|^2 \ , \tag{4.12}$$

$$V_S = |\sum_{n=1}^{p} c_n Z_n^3 \{1 + (S+\bar{S})\frac{f_{nS}}{f_n}\}|^2, \ \ V_T = \frac{1}{3}|\sum_{n=1}^{p} c_n Z_n^3\{3 + (T+\bar{T})\frac{dH_n/dT}{H_n}\}|^2. \tag{4.13}$$

For a minimum of the potential in the weak coupling region to exist, that is found to be a minimum of a zero energy, this have to be determined as a solution to the equations

$$\frac{\partial W}{\partial Y_n} = 0, \quad \frac{\partial W}{\partial T} = 0, \quad \frac{\partial W}{\partial S} = 0. \tag{4.14}$$

The first equation of (4.14) gives

$$\log\{Y_n^3 f_n(S) H_n(T)\} = -1 \Longrightarrow Y_n^3 = e^{-1} e^{-\frac{32\pi^2 k_n S}{c_n}} \frac{1}{H_n^i(T)}, \ i = 1, 2. \tag{4.15}$$

[10]obtained by integrating out the composite gaugino superfield Y_n.

That makes the truncated superpotential to take the form

$$W^{tr} = -\frac{e^{-1}}{32\pi^2} K(S) h(T), \tag{4.16}$$

where

$$K(S) = \sum_{n=1}^{p} c_n e^{-32\pi^2 \frac{k_n S}{c_n}}, \tag{4.17}$$

$$h(T) = \frac{1}{H_n^i(T)}, \ i = 1, 2. \tag{4.18}$$

Making use of the H_n's in eqn.'s (4.8, 4.9) we derive the allowed truncated superpotential forms when the source of non-perturbative effects is not specified respectively as

$$W^{tr\,(1)} = -\frac{e^{-1}}{32\pi^2} K(S) \frac{j^a(T)(j(T) - 1728)^b}{\eta^6}, \ a, b \ \in \ Z^{\dagger}, \tag{4.19}$$

or

$$W^{tr\,(2)} = -\frac{e^{-1}}{32\pi^2} K(S) \frac{1}{\eta(T)^6} \frac{1}{j^a(T)(j(T) - 1728)^b}, \ a, b \ \in \ Z^{\dagger}. \tag{4.20}$$

The superpotentials (4.19), (4.20) avoid and have singularities in the fundamental domain respectively. The potential corresponding to the superpotentials of eqn.'s (4.19, 4.20) does have a stable minimum with respect to the S-field. In particular the dilaton can be stabilized at the weakly coupled regime as we are using more than one condensates in the way suggested in the racetrack models of [37, 38]. The value of the T-moduli may be found by minimizing the potential arising from substituting the value of the auxiliary field Y in (4.10). Equivalently, we can use the values of the truncated superpotentials (4.19, 4.20) in (3.9) and minimize with respect to the dilaton and T moduli.

4.2 *With matter field condensates*

Consider a heterotic string compactification vacuum such that Wilson line background fields are associated to chiral matter fields A. The latter break the hidden sector gauge group to an $SU(N)$ hidden sector representing SQCD with M massive flavours[11] $Q \oplus \bar{Q}$, in the representations $N \oplus \bar{N}$, N being the number of flavours.

Recall again that the effective action in the usual supergravity was given in eqn. (4.1). In the presence of matter fields the effective superpotential as dictated by Ward identities and modular invariance has been written in [39] as

$$W^{matter} = \frac{1}{32\pi^2} Y^3 \log\{e^{32\pi^2 S}[(c\eta(T)]^6)^{N-M/3} Y^{3N-2M} det\Pi\} - tr A\Pi, \tag{4.21}$$

[11]Q represent the matter fields

Here, c is an unknown constant, A chiral matter fields associated with the Wilson line background fields and $\Pi_i^j = Q_i \bar{Q}^j$, $j = 1, \ldots, M$, represent the matter bound states. We would like now, given the constructions of $W^{non-pert}$ originating from eqn. (3.4), to see if we can modify somehow eqn. (4.21) to incorporate the unknown non-perturbative T-duality dynamics in all possible forms. The possible modifications of (4.21) read

$$W^{matter(1)} = \frac{1}{32\pi^2} Y^3 \log\{e^{32\pi^2 S}[(c\eta(T)]^6)^{N-M/3} \left(\frac{1}{j^a(T)(j(T)-1728)^b}\right)^{N-M/3} Y^{3N-2M} \times det\Pi\} - trA\Pi,\ a,b \ \in \ Z^{\dagger}, \tag{4.22}$$

or alternatively

$$W^{matter(2)} = \frac{1}{32\pi^2} Y^3 \log\{e^{32\pi^2 S}[(c\eta(T)]^6)^{N-M/3} \left(j^a(T)(j(T)-1728)^b\right)^{N-M/3} Y^{3N-2M} \times det\Pi\} - trA\Pi,\ a,b \ \in \ Z^{\dagger}. \tag{4.23}$$

Because at the weak coupling limit, $ReS \to \infty$, gravity decouples and the string model behaves like globally supersymmetric QCD, global supersymmetry is not broken and the minimum of the potential is found by the conditions

$$\frac{\partial W}{\partial Y} = \frac{\partial W}{\partial \Pi} = 0. \tag{4.24}$$

Solving eqn.'s (4.24) results in

$$\begin{aligned} \frac{1}{32\pi^2} Y_{(1)}^3 &= 32\pi^2 e^{M/N-1}[c\eta]^{2\frac{M}{N}-6}\{j^a(j-1728)^b\}^{1-\frac{M}{3N}}][detA]^{1/N} exp(-32\pi^2 S/N), \\ \Pi &= \frac{1}{32\pi^2} Y_{(1)}^3 A^{-1},\ a,b,\ \in\ Z^{\dagger}, \end{aligned} \tag{4.25}$$

or

$$\begin{aligned} \frac{1}{32\pi^2} Y_{(2)}^3 &= 32\pi^2 e^{M/N-1}[c\eta]^{2\frac{M}{N}-6}\{j^a(j-1728)^b\}^{1-\frac{M}{3N}}][detA]^{1/N} exp(-32\pi^2 S/N), \\ \Pi &= \frac{1}{32\pi^2} Y_{(2)}^3 A^{-1},\ a,b,\in\ Z^{\dagger}, \end{aligned} \tag{4.26}$$

respectively. We can now eliminate the auxiliary composite fields Y, Π in eqn's (4.22), (4.23), by substituting their values from (4.25), (4.26) respectively and derive the truncated superpotential W_{matter}^{tr}, which depends only on the S, T and A fields. The truncated superpotential reads for the two cases considered

$$W_{trunc}^{(1)} = \hat{\Omega}(S)\mathcal{K}(T)[detA]^{1/N} \tag{4.27}$$

$$\hat{\Omega}(S) = -N exp(-32\pi^2 S/N) \tag{4.28}$$

$$\mathcal{K}(T) = (32\pi^2 e)^{M/N-1}[c\eta(T)]^{2M/N-6}\mathcal{P}^{(k)}(j(T)),\ k = 1,2 \tag{4.29}$$

where

$$\mathcal{P}^{(1)} = j^a(j-1728)^b,\ a,b,\ \in\ Z^{\dagger}, \tag{4.30}$$

$$\mathcal{P}^{(2)} = \frac{1}{j^a(j-1728)^b},\ a,b,\ \in, Z^{\dagger}. \tag{4.31}$$

The superpotentials associated with (4.30), (4.31) represent candidate solutions for non-perturbative superpotentials in the presence of the "Wilson lines" A.

4.3 *Stability of gaugino condensate in the truncated formalism*

To test whether or not the new constructions $W^{non-pert}$ of eqn. (4.19) might be phenomelogically preferred over the eqn. (4.20) form[12], we will look at the stabilization conditions for the gaugino condensate. For convenience we look at the case without matter field condensates of subsection 4.1. For convenience we choose the Kähler potential in the form

$$\begin{aligned} K_{pert} &= -\log(S+\bar{S}) - 3\log((T+\bar{T}), \\ K &= K_{pert} - 3\log(1 - \frac{9}{\psi}e^{\frac{K_{pert}}{3}}(Y\bar{Y})^{1/3}), \end{aligned} \tag{4.32}$$

where[13] ψ a constant. The effective potential of the theory is given by

$$V = \frac{b}{6}\frac{|\lambda|^4}{(1-|\tilde{z}|^2)^2}\{3|1+\ln(c\mathcal{B}(S)H^k(T)e^{-K/2}\tilde{z}^3)|^2 + \mathcal{E}|\tilde{z}|^2\}, \tag{4.33}$$

where $\mathcal{B}(\mathcal{S})$ describes the gaugino condensation dynamics of the dilaton,

$$\tilde{z} = Y exp(K_{pert}/2), \tag{4.34}$$

$\tilde{z}$ the modified Z variable($Z = \tilde{z}$) of (4.11), $H^k(T)$ is taken from from (4.8) and (4.9) and

$$\mathcal{E} = (S+\bar{S})^2\left|-\frac{1}{(S+\bar{S})} + \frac{\mathcal{B}'}{\mathcal{B}}\right|^2 + \frac{1}{3}\left|3 + (T+\bar{T})\frac{H'(T)}{H(T)}\right|^2 - 3, \tag{4.35}$$

From the solution of eqn.'s (4.14), that describe the minimum of the potential at the weak coupling limit we can calculate the value of the gaugino condensate Y as

$$|<\lambda\lambda>| = |<Y>| = e^{-1/3}\frac{(c\mathcal{B}(S)H(T))^{-1/3}}{(T+\bar{T})^{1/2}},\ \ \tilde{z}_{min} = e^{-1/3}\frac{(c\mathcal{B}(S)H(T))^{-1/3}}{(S+\bar{S})^{1/6}(T+\bar{T})^{1/2}}. \tag{4.36}$$

[12]something similar may be shown in the two cases in (4.29), of $k=1$ over the $k=2$ case, when matter field condensates are included,

[13]Note that the effective action in this form was singled out in [43]. By setting $\psi = 9$ we recover the normalization of [36].

The stationary point of the potential (4.33), a zero energy minimum, is reached when $\mathcal{E} \to 0$ and in addition $S_R\mathcal{B}' - \mathcal{B} = 0$. The last condition is necessary for the potential to achieve a minimum at the weak coupling region [15]. In this case, in the absence of a well defined dilaton dynamics, a minimum at weak coupling could be easily achieved for the superpotential class of [44], $\mathcal{B}(S) = c' + h'_{\tilde{a}} e^{\frac{3S}{2b_{\tilde{a}}}}$, where $\tilde{a}$ counts the number of condensates, when the parameter c' is very small. Exactly as in [35] the minimum of the potential is found when the phase of the condensate is aligned in such a way that the quantity $(c\mathcal{B}(S)H^k(T)e^{-K/2}\tilde{z}^3)$ is real and positive.

Notice that we are interested in the minimum of the potential at weak coupling because phenomenology requires unification of the gauge couplings of the standard model at a unification scale of 10^{16} GeV. In particular(see for example [41]) this requires $S_R \approx 2$. The behaviour of the potential (4.33) as to whether it reaches its stationary point depends crucially on the behaviour of the parameter $\mathcal{I} \equiv \mathcal{E}|\tilde{z}|^2$.

Notice that we make here an important point. We require that in order for the potential to reach the zero energy minimum the parameter $\mathcal{I}$ has to vary smoothly at a general point of the moduli space. Since the dilaton dynamics is in general unknown, we assume that $S_R\mathcal{B}' - \mathcal{B}$ goes to zero smoothly. Separating the T-modulus dependence we find that

$$\mathcal{I} \sim (T + \bar{T}) \, \frac{|H'(T)|^2}{|H(T)|^2} \, H(T)^{-2/3}. \tag{4.37}$$

We rewrite the value of H from (4.8), (4.9) into the form

$$H(T) = \eta^6(T) \frac{1}{j^a (j - 1728)^b}, \quad a, b \, \in \, Z. \tag{4.38}$$

The only points in the moduli space that $\mathcal{I}$ may have a problem is exactly the self-dual, duality invariant points of the potential (4.36) $T = 1, \ \rho$. To illustrate the nature of singularities in the moduli space we notice that j has a zero of order 3 at $T = \rho$ and (j-1728) has a zero of order 2 at $T = 1$. That means

$$(j - 1728) \sim (T-1)^2 \; when \; T \; \to \; 1, \; j \; \sim \; (T - \rho)^3 \; when \; T \; \to \; \rho. \tag{4.39}$$

In addition,

$$\frac{H'}{H} \stackrel{T \to \rho}{\to} (T - \rho)^{-2}, \quad \frac{H'}{H} \stackrel{T \to 1}{\to} (T-1)^{-2} \; . \tag{4.40}$$

Before examining the effect of (4.38) to (4.37) let us produce the points where the potential corresponding to the class of superpotentials (2.10) blows up. As $T \to \rho$ we get [42] that (4.35) breaks down for $m \leq 2$. Notice that if we perform a similar analysis for the superpotentials (2.10) as $T \to 1$ we get that

$$\mathcal{I} \sim (T-1)^{-2+2m/3}, \tag{4.41}$$

That means that (4.36) breaks down[14], as $T \to 1$ for the range of parameters $n \leq 2$. Summarizing

$$V|_{j^{n/3}(J-1728)^{m/2}} \stackrel{T\to\rho}{\to} \infty, \;\; n \leq 2, \; V|_{j^{n/3}(J-1728)^{m/2}} \stackrel{T\to 1}{\to} \infty, \;\; m \leq 2. \tag{4.42}$$

In our case, as $T \to 1$, $\mathcal{I}$ changes as

$$\mathcal{I}|_{j^a(j-1728)^b} \sim (T-1)^{4b/3-2}. \tag{4.43}$$

Clearly, the values of b which the potential may avoid to develop a singularity as $T \to 1$ are

$$b \leq 1. \tag{4.44}$$

A similar analysis can be performed for (4.38) as $T \to \rho$. In this case

$$\mathcal{I}|_{j^a(j-1728)^b} \sim (T-\rho)^{2a-2}, \tag{4.45}$$

which means that $\mathcal{I}$ does not become infinite when a avoids the values

$$a \leq 0. \tag{4.46}$$

Note that the upper safe value $m = 3$ in (4.42) corresponds exactly to the value $b = 1$ of the superpotentials (4.38). That means that (4.38) naturally does not make the potential to breaks down as $T \to \rho$.

Thus we have derived the allowed (a, b) values for the potential to be finite[15] at every point in the moduli space so that the weak coupling minimum can be reached, to be

$$(a; b) = (1, 2, 3, ...; 2, 3, ...). \tag{4.47}$$

The "allowed" values (4.47) in the parameter space (a, b) constitute a criterion for the potential to avoid singularities in the fundamental domain. Notice, that for those values of (a, b) we should be able to find a minimum at weak coupling. In addition, it is possible in this case that the minimum is not necessarily at weak coupling because now the parameter $S_R\mathcal{B}' - \mathcal{B}$ is allowed to varied smoothly, with no T-dependence, and it is not necessary that it reaches zero.

It is worth noticing that for the minimum allowed values of a, b, $min(a, b) = (1, 2)$ an interesting possibility arises. For those values

$$W(T, S, a = 1, b = 2) = \mathcal{K}(S)\frac{1}{\eta^6(T)}\; j\; (j-1728)^2 \equiv \mathcal{K}(S)\frac{1}{\eta^6(T)}(j^3 - 3456j^2 + 1728^2 j). \tag{4.48}$$

[14] goes to infinity

[15] that demands the parameter $\mathcal{I}$ to vary smoothly over the whole moduli space

It is clear that (4.48) has an identical form to (2.19), the solution which can have a vanishing cosmological constant and broken supersymmetry at a generic point in the moduli space, for special values of the parameters l, m, n and broken supersymmetry,

$$W(T,S;min(a)=1,min(b)=2)=W^{j}(T,S)|_{l=-3456,m=1728^2,n=0}\quad . \tag{4.49}$$

Since the solutions (4.48) represent a particular basis for modular forms we expect (4.48) and (2.19) to be equivalent.

Note that despite that fact that the gauge kinetic function develops singularities at the self-dual points $T=1,\rho$ signalling the appearance of previously massive states becoming massless, the potential is finite at these points. In particular, the nature of the construction (3.7) is such that the potential naturally avoids to blow up at $T=\rho$ while at $T\rightarrow 1$, we enforce it to behave smoothly by restricting the $b\geq 2$. This is a novel feature of our potential (3.7) since the superpotentials (2.11) of [16] make the potential to become infinite when $(m,n)\leq 2$.

5 Conclusions

Because the degeneracy of the dilaton and the rest of the moduli fields have to be lifted by non-perturbative effects we studied the most general parametrizations of superpotentials such as the origin of non-perturbative effects is not specified in the context of $\mathcal{N}=1$ four dimensional heterotic string.

The dilaton dependence of superpotentials was examined only in the context of a $\mathcal{N}=1$ strong-weak coupling duality equivalence [21] of the $\mathcal{N}=1$ heterotic string actions. In this case, we derived the most general parametrizations of non-perturbative dynamics that do, or not, allow singularities in the fundamental domain generalizing results by [21, 28]. All available S-duality constructions[21, 28] are "included" in our construction.

Because classes of dilaton superpotentials like (3.11), tend to stabilize the dilaton at $S=1$, it is worth exploiting the possibility that our dilaton construction is generalized to subgroups of $SL(2,Z)_S$ such as those appearing in supersymmetric Yang-Mills [46, 29]. and in compactification of F-theory on K_3 surfaces when the Mordell-Weyl group is not trivial [30].

In another direction we found the most general parametrization of T-moduli dynamics, generalizing and complementing previous approach [16]. In fact what we did, was to allow for the most general parametrization of the corrections to the Kähler

class T-moduli in the gauge kinetic function f. We found that the nature of the superpotential, by construction, incorporates a novel criterion for avoiding singularities in the fundamental domain or places them outside the latter, e.g expressions (4.19), (4.20) respectively. To lowest order, in the parameters space (a, b), the finite scalar potential may correspond to vacua with vanishing cosmological constant and broken supersymmetry, dilaton auxiliary field not zero, found before [16] in a different context. Moreover, for this range of (a, b) parameters it is possible that the supersymmetry breaking minimum can break supersymmetry when the dilaton auxiliary field is not zero.

Anothr novel property of the superpotentials (3.8), for the perturbative heterotic string, is that the potential goes to to zero at the decompactification limit $T = \infty$.

Independently of what will be the exact form of the non- perturbative superpotential for $\mathcal{N} = 1$ heterotic string vacua, when we will be able to calculate it exactly, the superpotentials (2.10), captured the general quantitative structure of non-perturbative effects since they are constructed based on its modular properties. Our results have important consequences for supersymmetry breaking, CP violation, and inflation [45] in the context of $\mathcal{N} = 1$ four dimensional heterotic string theories. Such problems may require further study.

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